Minimizing the Variance of Cluster Mixture Models for Clustering Uncertain Objects

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Uncertainty

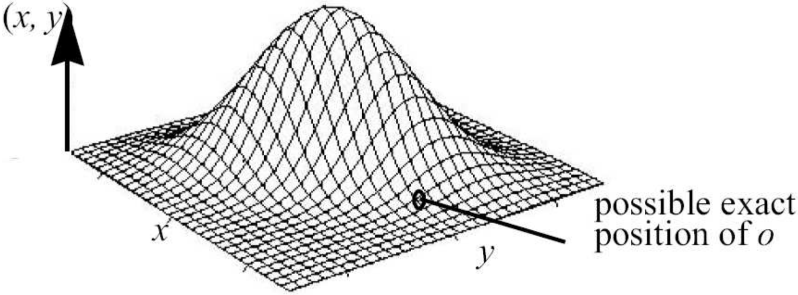
Uncertainty inherently affects data from a wide range of emerging application domains:

- sensor data
- location-based services (e.g., moving objects data)
- biomedical and biometric data (e.g., gene expression data)
- distributed applications
- RFID data
- ...

It is generally due to noisy factors, such as signal noise, instrumental errors, wireless transmission

Uncertain Objects (UO) (1)

Modeling by *regions (domains) of definition* and *probability density functions (pdfs)*



Uncertain Objects (UO) (2)

- *m*-dimensional region
- multivariate pdf defined over the region

Definition (uncertain object)

An uncertain object o is a pair $(\mathcal{R}, f)$:

- $\mathcal{R} \subseteq \Re^m$ is the m-dimensional in which o is defined
- $f : \Re^m \longrightarrow \Re_0^+$ is the probability density function of o at each point $\vec{x} \in \Re^m$ such that:

$$f(\vec{x}) = 0, \ \ \forall \vec{x} \in \Re^m \setminus \mathcal{R} \qquad \text{and} \qquad f(\vec{x}) > 0, \ \forall \vec{x} \in R$$

Clustering Uncertain Objects (1)

Major approaches:

- partitional clustering methods:
  - uncertain version of k-Means [Chau et Al., PAKDD'06] and its relative optimizations [Ngai et Al., ICDM'06, Lee et Al., ICDM Work.'07, Chui et Al., ICDM'08]
  - uncertain version of k-Medoids [Gullo et Al., SUM'08]
- density-based clustering methods:
  - uncertain version of DBSCAN [Kriegel and Pfeifle, KDD'05]
  - uncertain version of OPTICS [Kriegel and Pfeifle, ICDM'05]
- hierarchical clustering methods [Gullo et Al., ICDM'08]

Clustering Uncertain Objects (2)

Issues of existing algorithms:

1. they require some notion of distance between uncertain objects
   - hard task as existing notions are either inaccurate or inefficient
2. they generally suffer from efficiency issues
   - intrinsically due to the adopted formulations, which require to continuously execute critical operations

Minimizing Mixture Model Variances for Clustering Uncertain Objects

Goal: to solve both the issues arising from existing algorithms for clustering uncertain objects

Proposal

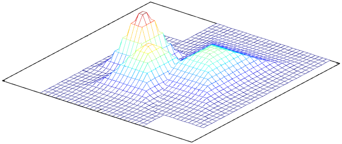
Novel formulation to the problem of clustering uncertain objects whose major features are:

1. exploiting *mixture models* for representing the clusters to be identified
2. employing the minimization of the *variance* of the mixture models as optimization criterion

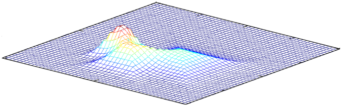
Cluster Mixture Models (Uncertain Prototypes)

Mixture model (uncertain prototype) of cluster C: $\mathcal{P}_C = (\mathcal{R}_C, f_C)$

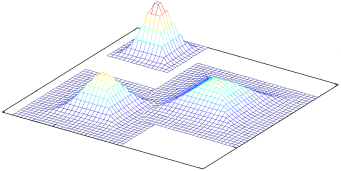
$$\mathcal{R}_C = \bigcup_{o=(\mathcal{R},f)\in C} \mathcal{R} \qquad f_C(\vec{x}) = (|C|)^{-1} \sum_{o=(\mathcal{R},f)\in C} f(\vec{x})$$



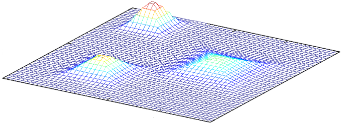
(a)



(b)



(c)



(d)

(a)–(c): *Sets of uncertain objects*

(b)–(d): *The corresponding mixture models*

Proposed Formulation

Idea: minimizing variance of cluster mixture models

$$J(\mathcal{C}) = \sum_{C \in \mathcal{C}} \sigma^2(\mathcal{P}_C)$$

- **accuracy**: the lower the variance, the higher the cluster compactness
- **efficiency**: capability of exploiting interesting analytical properties

Computing objective function J

- Moving object o from $C \in \mathcal{C}$ to $\widehat{C} \in \mathcal{C}$ leads to a new $\mathcal{C}' = \mathcal{C} \setminus (C \cup \widehat{C}) \cup (C' \cup \widehat{C}')$, where $C' = C \setminus \{o\}$, $\widehat{C}' = \widehat{C} \cup \{o\}$
- $J(\mathcal{C}')$ can be efficiently computed in $\mathcal{O}(m)$ as:

$$J(\mathcal{C}') = J(\mathcal{C}) - (\sigma^2(\mathcal{P}_C) + \sigma^2(\mathcal{P}_{\widehat{C}})) + (\sigma^2(\mathcal{P}_{C'}) + \sigma^2(\mathcal{P}_{\widehat{C}'}))$$

MMVar algorithm

Input: A set $\mathcal{D}$ of UO; the number k of output clusters
Output: A partition $\mathcal{C}$ of $\mathcal{D}$

```
compute μ(o), μ2(o), ∀o ∈ D
C ← randomPartition(D, k)
compute μ(P_C), μ2(P_C), ∀C ∈ C
v ← J(C)
repeat
    for all o ∈ D do
        let C ∈ C be the cluster s.t. o ∈ C
        C* ← arg min_Ĉ J_C(C, o, Ĉ)
        if C* ≠ C then
            v = J_C(C, o, Ĉ)
            recompute C by moving o from C to C*
            recompute μ(P_C), μ2(P_C), μ(P_C*), μ2(P_C*)
        end if
    end for
until no object in D is relocated
```

$$\begin{array}{l}
\text{compute } \vec{\mu}(o),\ \vec{\mu}_2(o),\ \forall o \in \mathcal{D} \\
\mathcal{C} \leftarrow \mathit{randomPartition}(\mathcal{D}, k) \\
\text{compute } \vec{\mu}(\mathcal{P}_C),\ \vec{\mu}_2(\mathcal{P}_C),\ \forall C \in \mathcal{C} \\
v \leftarrow J(\mathcal{C}) \\
\textbf{repeat} \\
\quad \textbf{for all } o \in \mathcal{D} \textbf{ do} \\
\quad\quad \text{let } C \in \mathcal{C} \text{ be the cluster s.t. } o \in C \\
\quad\quad C^* \leftarrow \arg\min_{\widehat{C}} J_{\mathcal{C}}(C, o, \widehat{C}) \\
\quad\quad \textbf{if } C^* \neq C \textbf{ then} \\
\quad\quad\quad v = J_{\mathcal{C}}(C, o, \widehat{C}) \\
\quad\quad\quad \text{recompute } \mathcal{C} \text{ by moving } o \text{ from } C \text{ to } C^* \\
\quad\quad\quad \text{recompute } \vec{\mu}(\mathcal{P}_C), \vec{\mu}_2(\mathcal{P}_C), \vec{\mu}(\mathcal{P}_{C^*}), \vec{\mu}_2(\mathcal{P}_{C^*}) \\
\quad\quad \textbf{end if} \\
\quad \textbf{end for} \\
\textbf{until} \text{ no object in } \mathcal{D} \text{ is relocated}
\end{array}$$

- MMVar converges to a local optimum of function J in a finite number I of iterations
- MMVar works in $\mathcal{O}(I\ k\ |\mathcal{D}|\ m)$

Evaluation Methodology

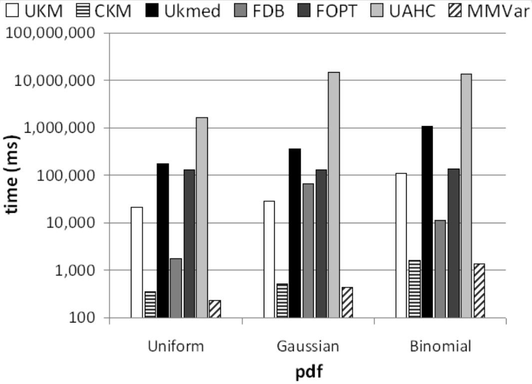
- Benchmark datasets from UCI (Iris, Wine, Glass, Ecoli, Yeast, Image, Abalone, Letter)
- Uncertainty generated **synthetically** and modeled according to *Uniform* (U), *Normal* (N), and *Binomial* (B) pdfs
- Evaluation in terms of:
  - **accuracy** (w.r.t. reference classifications according to *F-Measure*)
  - **efficiency**
- Competitors: UK-means (UKM), CK-means (CKM), UK-medoids (UKmed), $\mathcal{F}$DBSCAN ($\mathcal{F}$DB), $\mathcal{F}$OPTICS ($\mathcal{F}$OPT), U-AHC

Accuracy Results

		F-measure ($F \in [0,1]$)						
data	*pdf*	UKM	CKM	UKmed	$\mathcal{F}$DB	$\mathcal{F}$OPT	UAHC	**MMVar**
avg score	U	0.601	0.675	0.729	0.331	0.575	0.626	**0.731**
	N	0.54	0.582	0.493	0.441	0.475	0.606	**0.657**
	B	0.476	0.363	0.602	0.295	0.525	0.508	**0.716**
overall avg. score		0.539	0.54	0.608	0.356	0.525	0.58	**0.701**
overall avg. gain		0.162	0.161	0.093	0.345	0.176	0.121	—

- MMVar achieved the best overall scores, from +0.093 (w.r.t. UKmed) to +0.345 (w.r.t. $\mathcal{F}$DB)
- MMVar achieved the best avg scores on all the pdfs
  - maximum avg gain of 0.254 (Binomial)
  - minimum avg gain of 0.134 (Normal)

Efficiency Results



- MMVar performed faster than CKM
- MMVar drastically outperformed all other competitors but CKM (at least 1 order of magnitude, up to 5 orders)
- Slowest methods: UAHC and UKmed; fastest methods: CKM and $\mathcal{F}$DB

Conclusion

- Novel formulation to the problem of clustering uncertain objects
  - Cluster mixture models
  - Minimization of the variance of mixture models
- MMVar heuristic algorithm
- Significant advantages achieved by MMVar in terms of efficiency and accuracy w.r.t. existing algorithms

Thanks!

Datasets

dataset	*# objects*	*# attributes*	*# classes*
Iris	150	4	3
Wine	178	13	3
Glass	214	10	6
Ecoli	327	7	5
Yeast	1,484	8	10
Image	2,310	19	7
Abalone	4,124	7	17
Letter	7,648	16	10